

FLOW IN A LAMINAR BOUNDARY LAYER UNDER
INTENSIVE INJECTION AND RADIANT
HEAT EXCHANGE

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An asymptotic solution has been obtained for the system of boundary layer equations with intense injection of a foreign absorbing substance for optically thick and thin boundary layers taking account of the influence of a magnetic field. Analytical formulas are presented which permit computation of the temperature, concentration, and heat flux profiles.

The system of laminar boundary layer equations for radiating gas flow around a plane or axisymmetric body is examined. The foreign absorbing gas, driving back the external stream, is injected intensively through the body surface. It is considered that the injection intensity is not too great so that the pressure does not change across the boundary layer. In such case, the flow is described by the system of boundary layer equations which is for a binary mixture without taking account of the work of the pressure forces, the Joulean and viscous dissipation, the chemical reactions, and under the assumption that the specific heat depends slightly on the temperature [1-3]:

$$\begin{aligned} \frac{\partial}{\partial x} \rho u r_0^k + \frac{\partial}{\partial y} \rho v r_0^k &= 0, \\ \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \mu \frac{\partial u}{\partial y} - \sigma u B^2, \\ \rho u c_p \frac{\partial T}{\partial x} + \rho v c_p \frac{\partial T}{\partial y} &= \frac{\partial}{\partial y} \kappa_m \frac{\partial T}{\partial y} + \frac{\partial q_r}{\partial y}, \\ \rho u \frac{\partial c}{\partial x} + \rho v \frac{\partial c}{\partial y} &= \frac{\partial}{\partial y} \rho D \frac{\partial c}{\partial y}. \end{aligned} \quad (1)$$

Indeed, under intensive injection the influence of viscosity is manifested in the neighborhood of the junction of the streams in the domain $\delta \sim L/\sqrt{Re_\infty}$.

The thickness of the internal flow domain Δ can be determined from the continuity equation $u/L \sim v_W/\Delta$ and the equality of the pressures on the interface $\rho_W u^2 \sim \rho_\infty U_\infty^2$.

The influence of the viscosity will be essential only in a small domain as compared with the whole inner zone ($\delta \ll \Delta$) if $1/\sqrt{Re_\infty} \ll (v_W/U_\infty)\sqrt{\rho_W/\rho_\infty}$. At the same time, the injection cannot be arbitrarily great since it follows from the Navier-Stokes equations that $\partial p/\partial y \sim \Delta/L \cdot \partial p/\partial x$.

Therefore, if $\Delta^2 \ll L^2$, meaning $\rho_W v_W^2 \ll \rho_\infty U_\infty^2$, then the pressure change across the boundary layer cannot be taken into account and the system of equations takes the form (1).

The system (1) is supplemented by the following boundary conditions

$$u = 0, v = v_w, T = T_w, c = c_w \text{ for } y = 0, u = u_e, T = T_\infty, c = c_\infty \text{ for } y \rightarrow \infty. \quad (2)$$

The velocity on the outer edge of the boundary layer $u_e = Ax$ is indeed determined from the solution of a problem about external inviscid flow. These solutions are known for incompressible fluid flow around a cylinder or sphere in the absence of a magnetic field [4]. The question of the change in the velocity gradient A on the outer edge of the boundary layer in the presence of a magnetic field with lines of force in the

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free stream direction has been studied in [5, 6]. In the opinion of the authors of [5, 6], the constant A in the presence of a field is related with the corresponding value A_0 in the absence of a field by the relationship $A = A_0 \sqrt{1 + S}$.

By using self-similar transformations [2, 3], which are in the neighborhood of the forward stagnation point of a plane or axisymmetric body

$$\xi = \frac{\rho_\infty \mu_\infty A}{2 + 2k} x^{2+2k}, \quad \eta = \sqrt{\frac{A}{\Lambda v_\infty}} \int_0^y \frac{\rho}{\rho_\infty} dy, \quad (3)$$

and the assumption that $\rho\mu = \rho_\infty\mu_\infty$, the Eqs. (1) reduce to a system of ordinary differential equations

$$\frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} + \Lambda \left\{ \frac{\rho_\infty}{\rho} \left[1 + S \left(1 - \frac{\sigma}{\sigma_\infty} \frac{df}{d\eta} \right) \right] - \left(\frac{df}{d\eta} \right)^2 \right\} = 0, \quad (4)$$

$$\frac{d}{d\eta} \cdot \frac{1}{Pr_m} \cdot \frac{d\theta}{d\eta} + f \frac{d\theta}{d\eta} + \frac{dQ}{d\eta} = 0, \quad (5)$$

$$\frac{d}{d\eta} \cdot \frac{1}{S_c} \cdot \frac{dC}{d\eta} + f \frac{dC}{d\eta} = 0, \quad (6)$$

where

$$\theta = \frac{T - T_w}{T_\infty - T_w}; \quad C = \frac{c_w - c}{c_w - c_\infty}; \quad f = -\frac{\rho v}{\rho_\infty} \sqrt{\frac{\Lambda}{A \gamma_\infty}}; \quad Q = \frac{q_r}{\rho_\infty c_p (T_\infty - T_w)} \sqrt{\frac{\Lambda}{A v_\infty}}.$$

The boundary conditions (2) become

$$f = f_w, \quad \frac{df}{d\eta} = C_w = \theta_w = 0 \quad \text{for } \eta = 0, \quad (7)$$

$$\frac{df}{d\eta} = C_\infty = \theta_\infty = 1 \quad \text{for } \eta \rightarrow \infty.$$

The system of Eqs. (4)-(6) with the boundary conditions (7) without taking account of a magnetic field was solved numerically for a hypersonic flow in [7-9], and for subsonic flow with an external source of radiation and injection of an incompressible fluid, in [10]. An asymptotic solution has been obtained in [11, 12] for the system (4), (5) without taking account of radiation and a magnetic field under intensive injections ($-f_w \gg 1$). An asymptotic solution of the system (4)-(6) for intensive injection of a foreign absorbing gas is presented below taking account of the influence of a magnetic field.

1. Following [12] let us introduce the variables

$$Z = \left(\frac{df}{d\eta} \right)^2, \quad F = \frac{f}{f_w}.$$

Then the system (4)-(6) becomes

$$\frac{1}{f_w^2} \sqrt{Z} \frac{d^2 Z}{dF^2} + F \frac{dZ}{dF} + 2\Lambda \left\{ \frac{\rho_\infty}{\rho} \left[1 + S \left(1 - \frac{\sigma}{\sigma_\infty} \sqrt{Z} \right) \right] - Z \right\} = 0,$$

$$\frac{1}{f_w^2} \cdot \frac{d}{dF} \left(\frac{1}{Pr_m} \sqrt{Z} \frac{d\theta}{dF} \right) + F \frac{d\theta}{dF} + \frac{1}{f_w} \frac{dQ}{dF} = 0, \quad (8)$$

$$\frac{1}{f_w^2} \cdot \frac{d}{dF} \left(\frac{1}{S_c} \sqrt{Z} \frac{dC}{dF} \right) + F \frac{dC}{dF} = 0.$$

The boundary conditions are

$$Z = \theta = C = 0 \quad \text{for } F = 1; \quad Z = \theta = C = 1 \quad \text{for } F \rightarrow \infty. \quad (9)$$

Under intensive injection ($-f_w \gg 1$) the solution of the system (8) can be sought as a power series in the parameter $1/f_w$. To the accuracy of terms on the order of $1/f_w$, there follows from (8)

$$F \frac{dZ}{dF} + 2\Lambda \left\{ \frac{\rho_\infty}{\rho} \left[1 + S \left(1 - \frac{\sigma}{\sigma_\infty} \sqrt{Z} \right) \right] - Z \right\} = 0, \quad (10)$$

$$F \frac{d\theta}{dF} = 0, \quad F \frac{dC}{dF} = 0. \quad (11)$$

Equations (11) and the boundary conditions (9) permit the assertion that the temperature and concentration are constant everywhere outside the domain of the stream junction, where terms with higher derivatives turn out to be essential in the general case. Therefore, for $-f_w \rightarrow \infty$ in the whole inner domain ($f < 0$) it can be considered that $\theta = C = 0$, i.e., the gas temperature equals the wall temperature, and the concentration equals the concentration of the injected gas, in the outer domain ($f > 0$), analogously we have $\theta = C = 1$.

Furthermore, the boundary condition $Z = 0$ for $F = 1$ should be chosen in seeking the stream function.

As is shown below, neglecting the higher derivatives does not result in noticeable error in the whole internal domain, with the exception of a small neighborhood of the stream junction. Then, under the condition that the electrical conductivity in the whole inner zone is small ($\sigma_w \ll \sigma_\infty$), we obtain from (10)

$$Z = \gamma_0^2 (1 - F^{2\Lambda}), \quad \left(\gamma_0^2 = \frac{\rho_\infty}{\rho_w} (1 + S) \right).$$

Therefore

$$f = f_w \cos \frac{\gamma_0 \eta}{f_w} \quad \text{for } \Lambda = 1; \quad f = f_w - \frac{\gamma_0^2 \eta^2}{4f_w} \quad \text{for } \Lambda = 0.5. \quad (12)$$

From (12) it follows from $\rho_\infty (1 + S) = \rho_w$ that $df/d\eta = 1$ at the stream junction ($\eta = \lambda$).

In the external domain ($\eta > \lambda$), the solution of the motion equation

$$\frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} + \Lambda \left[1 - \left(\frac{df}{d\eta} \right)^2 \right] = 0 \quad (13)$$

with the boundary conditions

$$f = 0, \quad \frac{df}{d\eta} = 1 \quad \text{for } \eta = \lambda, \quad \frac{df}{d\eta} = 1 \quad \text{for } \eta \rightarrow \infty, \quad (14)$$

should be sought.

Evidently the solution of (13) satisfying the boundary conditions (4) is

$$f = \eta - \lambda. \quad (15)$$

If $\rho_\infty (1 + S) \neq \rho_w$, then a domain where the transition from $df/d\eta$ defined by (12) to the value $df/d\eta = 1$ at $\eta \rightarrow \infty$ occurs, exists in the neighborhood of $\eta = \lambda$. Seeking the solution in the neighborhood of the stream junction is made complicated by the dependence of ρ and σ on θ and C . As is seen from (5), (6), the main change in the temperature and concentration occurs at the distances $\eta - \lambda \sim 1/\sqrt{\text{Pr}}$ and $\eta - \lambda \sim 1/\sqrt{\text{Sc}}$, respectively.

Putting $\lambda \gg \max(1.1/\sqrt{\text{Pr}}, 1/\sqrt{\text{Sc}})$, it can be considered in determining λ in a first approximation that the stream function in the whole internal domain is described by (12) and in the external domain by (15).

Therefore,

$$\lambda = -\beta f_w / \gamma_0 \quad (\beta = \frac{1}{2}\pi \quad \text{for } \Lambda = 1, \quad \beta = 2 \quad \text{for } \Lambda = 0.5). \quad (16)$$

It follows from (16) that the boundary of the standoff λ in self-similar coordinates diminishes as the magnetic interaction parameter grows for $\sigma_w \ll \sigma_\infty$.

It should be noted that the stream function described by (12), (15) for $\Lambda = 0.5$, $\rho = \rho_\infty$, $S = 0$ is an exact solution of (4). For $\Lambda = 1$ neglecting the highest derivative in (4) results in an error in the internal domain

on the order of

$$\frac{1}{f_w^2} \sqrt{\frac{\rho_\infty}{\rho_w}} \left(\sin \frac{\eta}{f_w} \sqrt{\frac{\rho_\infty}{\rho_w}} \right) / \cos^2 \frac{\eta}{f_w} \sqrt{\frac{\rho_\infty}{\rho_w}}.$$

Hence, it follows that the influence of viscosity turns out to be negligible near the wall and is manifested only in the neighborhood of the stream junction in the domain $|\eta - \lambda| \leq (\rho_w/\rho_\infty)^{1/4}$.

2. In the radiation heat conduction approximation [13-15], heating of unit volume because of radiation dq_r/dy can be combined with the heat influx due to molecular heat conduction, and it can be considered that $Pr \ll 1$. The condition for applicability of this approximation is $l(d\eta/dy) \ll 1/\sqrt{Pr}$, where l is the radiation mean free path, and $dy/d\eta \cdot 1/\sqrt{Pr}$ is the characteristic scale within which the temperature changes.

If the temperature θ_λ at the stream junction is considered known, then it follows from (5) that the temperature gradient at the wall equals

$$\left(\frac{d\theta}{d\eta} \right)_w = \theta_\lambda / \int_0^\lambda \exp \left[-Pr \int_0^\eta f(\eta') d\eta' \right] d\eta. \quad (17)$$

Assuming that $Pr f_w^2 \gg 1$ and the stream function is determined by (12) in the whole domain $0 < \eta < \lambda$, we can obtain

$$\left(\frac{d\theta}{d\eta} \right)_w = 2\theta_\lambda \sqrt{\frac{Pr\gamma_0}{2}} \exp \left(-\frac{\beta_1 Pr f_w^2}{\gamma_0} \right), \quad (18)$$

where $\beta_1 = 1$ for $\Lambda = 1$; $\beta_1 = 4/3$ for $\Lambda = 0.5$.

The location of the stream junction is determined with a relative error on the order of $(1/\lambda\sqrt{Pr\gamma_0})(1 - \gamma_0/\gamma)$, where γ is the derivative of the stream function at the stream junction, defined in conformity with (4), (5). This error originates because the stream function in the domain $\eta - \lambda \sim 1/\sqrt{Pr}$ has a slope γ different from the γ_0 calculated by means of (12) because of the change in density and viscosity.

The exponent of the exponential in (18) is determined to the accuracy of terms on the order of $1/2(1 - \gamma_0/\gamma)$.

A model assuming piecewise-constant properties of the medium is too rough to compute the temperature θ and concentration C because the main change in the quantities θ and C occurs in the neighborhood of the stream junction where it is necessary to know the exact behavior of the stream function in the general case.

If the change in density is due mainly to the change in temperature, then as is seen from (4) and (5), for $Pr \ll 1$ it should be considered that

$$f = \gamma(\eta - \lambda) \left(\gamma^2 = \frac{\rho_\infty}{\rho_\lambda} \left[1 + S \left(1 - \frac{\sigma_\lambda}{\sigma_\infty} \right) \right] \right) \quad (19)$$

in the domain of greatest change in the temperature ($|\eta - \lambda| \leq 1/\sqrt{Pr}$).

In the general case, the change in the stream function in the neighborhood of the stream junction is due to the change in density with temperature and to the influence of viscosity.

A computation of the temperature by using the stream function in the form (19) shows that

$$\theta = \frac{\Phi(\lambda\sqrt{Pr\gamma}) + \Phi[(\eta - \lambda)\sqrt{Pr\gamma}]}{1 + \Phi(\lambda\sqrt{Pr\gamma})} \quad (20)$$

$$\left(\Phi(x) = \frac{2}{\sqrt{2\pi}} \int_0^x \exp\left(-\frac{t^2}{2}\right) dt \right).$$

Therefore the dependence of the density on the coordinate η is determined by the argument $(\eta - \lambda)\sqrt{Pr\gamma}$.

As has been shown earlier, the influence of the viscosity is manifested in a considerably smaller domain $|\eta - \lambda| \leq 1$. Outside the viscous boundary layer domain, where the main change in density $1 < |\eta - \lambda| < 1/\sqrt{Pr\gamma}$ occurs, the order of magnitude of the discarded terms in (4) can be estimated. Assuming that

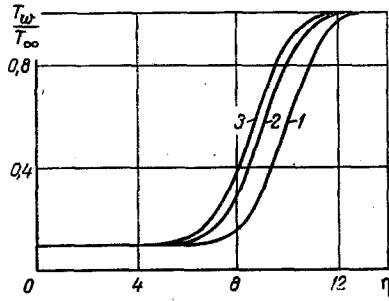


Fig. 1. Temperature distribution for the case $T_w/T_\infty = 0.1$, $\Lambda = 1$, $f_w = -2$: 1) by (23) (first approximation); 2) by (23) (second approximation); 3) by [12].

In conformity with (18) and (21), the magnitude of the heat flux can be determined approximately as follows:

$$q = \kappa_w (T_\infty - T_w) \frac{\rho_w}{\rho_\infty} \sqrt{\frac{A \text{Pr} \gamma_0}{2\pi \nu_\infty \Lambda}} \exp\left(-\frac{\beta_1}{\gamma_0} \text{Pr} f_w^2\right). \quad (22)$$

It follows from (22) that as the injection parameter grows, as does also the Prandtl number of the injected gas, the heat flux at the wall decreases exponentially and if $\sigma_\lambda \ll \sigma_\infty$, then taking into account that $f_w^2 \sim \sqrt{1+S}$, will be independent of the magnetic interaction parameter S .

Results of a computation by means of (20) for the case $\text{Pr} = 1$, $T_w/T_\infty = 0.1$, $\Lambda = 1$, $f_w = -2$ are shown in Fig. 1. The model of piecewise-constant properties which yields a somewhat exaggerated value of λ was used in a first-approximation calculation. Taking into account the temperature distribution obtained, and therefore, the density also, the value of λ in a second approximation is determined by means of (16) with ρ_w replaced by $\langle \rho \rangle$:

$$\langle \rho \rangle = \frac{1}{\lambda} \int_0^\lambda \rho d\eta.$$

Even in the case when the conditions $\text{Pr} \ll 1$, $-f_w \gg 1$ are not satisfied, a temperature computation by means of (20) agrees satisfactorily, as is seen from Fig. 1, with the results obtained earlier [12].

3. If the radiation mean free path is commensurate with or exceeds the dimension of the internal domain ($l \gtrsim \Delta$), then heating of a gray gas because of radiation for the one-dimensional case and a black wall is determined, as is known, by a known integral relationship [13, 14].

Under intensive injection of a weakly absorbing gas, the temperature distribution in the inner zone is due mainly to radiation and convection. Molecular heat conduction exerts influence on the temperature distribution only in the stream junction domain. In this case, the main temperature drop occurs at the distances $|\eta - \lambda| \sim 1/\sqrt{\text{Pr}_m}$ ($\text{Pr}_m \sim 1$). It is assumed that heating of the injected gas and diffusion in the domain of the stream junction do not alter the radiation mean free path substantially in the internal domain.

If the radiation mean free path satisfies the condition $l(d\eta/dy) \gg 1/\sqrt{\text{Pr}_m}$, then radiation from the wall and from the gas into the whole internal domain can be neglected. Then the above-mentioned integral relationship becomes

$$\frac{dq_r}{dy} = 2\alpha\sigma_0 T_* E_2(\tau_w - \tau), \left(E_n(x) = \int_1^\infty \exp(-xt) \frac{dt}{t^n} \right),$$

where T_* is the radiation temperature which can be set equal to T_λ since it is considered that the radiation mean free path in the external domain satisfies the condition $l(d\eta/dy) \ll 1/\sqrt{\text{Pr}_r}$ ($\text{Pr}_r \ll 1$). The condition of equality of the normal heat flux components on the interface ($\eta = \lambda$) means that the heat arriving from the external domain of radiation heat conduction at the inner zone is transported by the molecular heat conduction and radiation:

the stream function in this domain corresponds to (19) in order of magnitude, it should be considered that the exact solution of (4) will depend on the same argument as the density, i.e., $(\eta - \lambda)\sqrt{\text{Pr}\gamma}$. Furthermore, since $f \sim \gamma(\eta - \lambda)$, $df/d\eta \sim \gamma$, then $d^2f/d\eta^2 \sim \gamma\sqrt{\text{Pr}\gamma}$, $d^3f/d\eta^3 \sim \text{Pr}\gamma^2$. Then in the domain under consideration

$$\frac{d^3f}{d\eta^3} / \left(\frac{df}{d\eta} \right)^2 \sim \text{Pr}, \quad f \frac{d^2f}{d\eta^2} / \left(\frac{df}{d\eta} \right)^2 \sim (\eta - \lambda) \sqrt{\text{Pr}\gamma}.$$

Hence, it is seen that in the domain of greatest change in temperature the stream function in (4) is almost linear with a coefficient γ determined in conformity with (19).

It follows from (20) that for intensive injections $\lambda\sqrt{\text{Pr}_\lambda\gamma} \gg 1$ at the interface

$$\theta(\lambda) = \frac{1}{2}, \quad \left(\frac{d\theta}{d\eta} \right)_\lambda = \sqrt{\frac{\text{Pr}_\lambda\gamma}{2\pi}}. \quad (21)$$

$$\kappa \left(\frac{dT}{dy} \right)_{\lambda+0} = \sigma_0 T_\lambda^4 + \kappa_m \left(\frac{dT}{dy} \right)_{\lambda-0}. \quad (23)$$

Since $\kappa_r \gg \kappa_m$ then the heat transport because of molecular heat conduction can be neglected in the boundary condition (23).

Indeed, $\kappa_m (dT/dy)_{\lambda-0} / \kappa_r (dT/dy)_{\lambda+0} \sim \sqrt{\kappa_m} / \kappa_r$. Then condition (23) becomes

$$\kappa_r \left(\frac{d\theta}{d\eta} \right)_{\lambda+0} = \frac{\sigma_0 T_\lambda^4}{T_\infty - T_w} \left(\frac{dy}{d\eta} \right)_\lambda \left(\left(\frac{dy}{d\eta} \right)_\lambda = \frac{\rho_\infty}{\rho_\lambda} \sqrt{\frac{v_\infty \Lambda}{A}} \right). \quad (24)$$

The solution in the internal domain ($\eta > \lambda$) can be found by considering the stream function linear (19), and the temperature on the boundary to be equal to θ_λ :

$$\theta = \theta_\lambda + (1 - \theta_\lambda) \Phi[(\eta - \lambda) \sqrt{\text{Pr}_r \gamma}].$$

Hence it follows that

$$\left(\frac{d\theta}{d\eta} \right)_{\lambda+0} = 2(1 - \theta_\lambda) \sqrt{\frac{\text{Pr}_r \gamma}{2\pi}}.$$

Considering $\kappa_r = 16/3 \sigma_0 T_\lambda^3$, we obtain an equation to determine θ_λ from the boundary condition (24):

$$\theta_\lambda = \frac{32/3 \varepsilon/l \sqrt{2\pi} - T_w/(T_\infty - T_w)}{1 + 32/3 \varepsilon/l \sqrt{2\pi}} \left(\varepsilon = l \left(\frac{d\eta}{dy} \right)_\lambda \sqrt{\text{Pr}_r \gamma} \right). \quad (25)$$

Hence, it is seen that as ε increases, the temperature on the interface θ_λ rises. However, the condition for applicability of the radiation heat conduction approximation demands that $\varepsilon \ll 1$. Hence θ_λ cannot exceed the value $\theta_\lambda \approx 0.8$ corresponding to $\varepsilon = 1$. To solve (25) it is necessary to know the dependence of ε on θ_λ . For small values of θ_λ the condition $\varepsilon \ll 1$ may not be satisfied.

Neglecting molecular heat conduction in the whole internal domain, the energy equation can be represented as

$$-l \frac{d\theta}{d\eta} = k E_2 \left[\tau_w \left(1 - \frac{\eta}{\lambda} \right) \right] \left(k = \frac{2\Lambda \alpha \sigma_0 T_\lambda^4}{\rho c_p A (T_\infty - T_w)} \right). \quad (26)$$

The temperature distribution near the surface ($\eta^2 \ll \lambda^2$) can be obtained from (26):

$$\theta = - \frac{\lambda k}{\tau_w l} \left\{ E_3 \left[\tau_w \left(1 - \frac{\eta}{\lambda} \right) \right] - E_3(\tau_w) \right\}.$$

The heat flux on the wall is determined not only by the radiation flux but also by heat transport by molecular heat conduction from the gas heated by radiation:

$$q = 2\sigma_0 T_\lambda^4 \left[E_3(\tau_w) + \frac{\alpha \kappa_w}{\rho_w c_p v_m} E_2(\tau_w) \right]. \quad (27)$$

It is seen from (27) that heat transport because of molecular heat conduction diminishes by a power law as the injection grows. The total heat flux depends essentially on the temperature on the stream junction boundary.

NOTATION

ρ	is the density;
c_p	is the specific heat;
μ	is the dynamic viscosity;
L	is the characteristic length scale;
κ_m, κ_r	are the coefficients of molecular and radiant heat conduction;
D	is the coefficient of diffusion of a binary mixture;
u, v	are the longitudinal and normal velocity components, respectively;
p	is the pressure;
T	is the temperature;
c	is the concentration;

q_r	is the radiation flux;
B	is the induction of the magnetic field whose lines of force are in the free stream direction (the magnetic Reynolds number is considered small);
σ	is the electrical conductivity;
x	is the distance along the surface from the forward stagnation point of the body;
y	is the distance along the normal;
r_0	is the radius of transverse curvature of the body surface;
$k = 0, \Lambda = 1$	is the plane case;
$k = 1, \Lambda = 0.5$	is the axisymmetric case;
$Re = \rho UL / \mu$	is the Reynolds number;
$Pr = \mu c_p / \kappa$	is the Prandtl number;
$Sc = \nu / D$	is the Schmidt number;
$S = \sigma_\infty B^2 / \rho_\infty A$	is the Stuart number;
σ_0	is the Stefan-Boltzmann constant;
α	is the absorption coefficient;
l	is the radiation mean free path;
$\tau = \alpha y$	is the optical thickness.

Subscripts

w	is the wall;
λ	is the stream junction point;
∞	is the outside the boundary layer.

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